Uniform Circular Motion

Introductory experiment

The concept UCM

Definition: A uniform circular motion is a motion in which an object moves in the same direction, with a constant speed along a circular path.

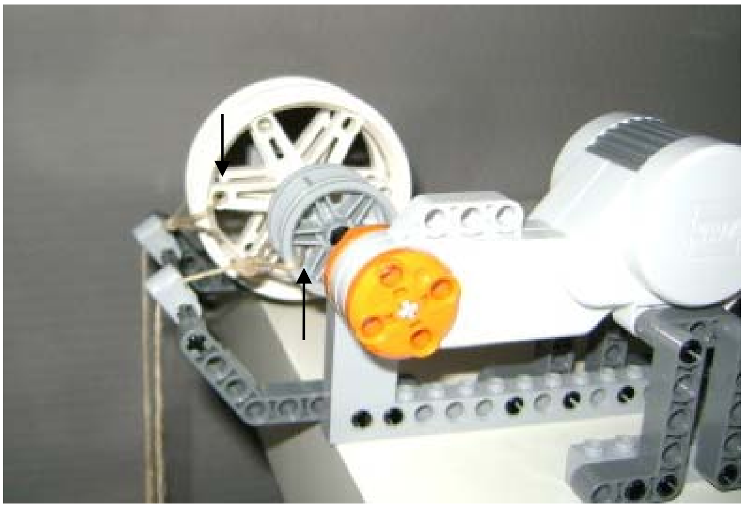
Do you know some examples of a UCM.

- ______________________________________
- ______________________________________
- ______________________________________

Experiment

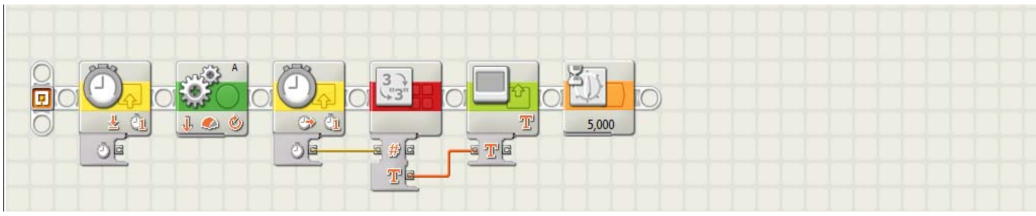
Goal: Demonstrate the relation between the peripheral speed and the angular speed.

Set-up: With this experiment we want to find the relation between the peripheral speed and the angular speed. Use the set-up shown below. The building plan will be provided by your teacher. The weights that are used weigh about 55 gr. The rope has a length of about 1m and is fixed to the spokes of the wheels. (see black arrows in the main picture)



<u>Program:</u> We want the engine to turn 3 rotations at an average power of 75%. Next, the engine stops. The time will be shown on the NXT display.

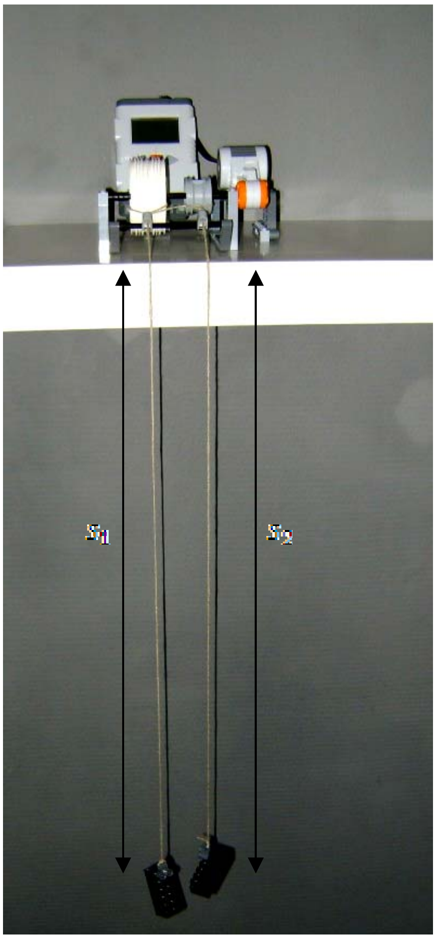
Which block do you use?	Which are the settings you apply?
timer	*timer: 1* *action: reset*
motor	*duration : 3 rotations* *power: 75%* *motor: A*
timer	*timer: 1* *action: read*
number in text	
display	*action: text*
wait	*control: time* *until: 5 seconds*



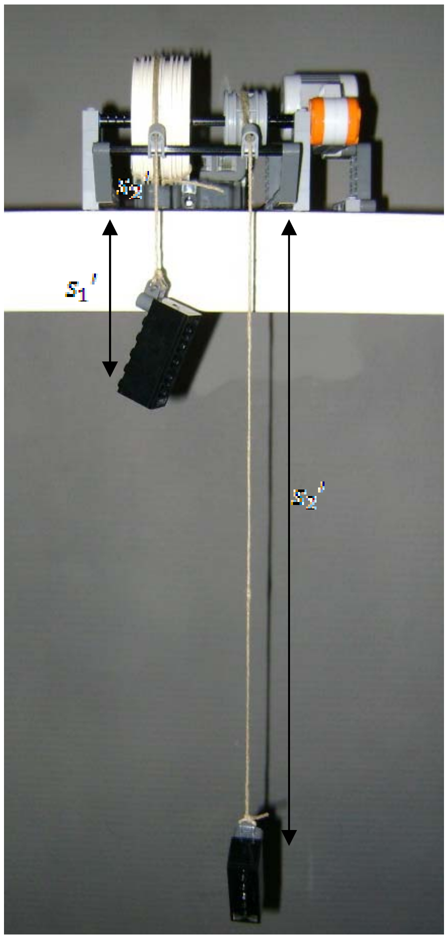
<u>Part 1:</u> In this experiment the diameter of the wheels will be important. The big white wheel will be called wheel 1 and the small grey wheel will be called wheel 2.

$d_{wheel1} = 0,0685 \quad m$ $\qquad$ $d_{wheel2} = 0,0295 \quad m$

<u>Part 2:</u> Let the program run. The shaft will turn 3 times. Then a time will be shown on the NXT display. This is the time needed for the NXT to make the 3 rotations.
To determine the displacement of the weights, you measure the distance from the weights to the table twice. Once before and once after the experiment. The difference between the two measurements is the displacement of the weight. This is illustrated in the pictures below.



Pic1.1 UCM: Position **before** the experiment



Pic1.2 UCM: Position **after** the experiment

Measure the displacement of each of the two weights. Fill in the time that was shown on the NXT display in the following table. After each execution of the experiment, slowly pull down the weights in order to start again.

Measurement	Time $[ms]$
1st execution	2634
2and execution	2598
3de execution	2617
average t $[s]$	2,6

Measurement	displacement wheel 1 $s = s_1 - s_{1'}$ $[m]$	displacement wheel 2 $s = s_2 - s_{2'}$ $[m]$
1st execution	0,60	0,27
2and execution	0,57	0,27
3de execution	0,56	0,28
average s $[m]$	0,58	0,27

Determine the speed of each weight. $v = s / t$

Wheel 1

weight 1 moves 0,58 m in 2,6 s, so $v_1 = \frac{0.58}{2.6} \left[\frac{m}{s}\right] = 0{,}223\ m/s$

Wheel 2

weight 2 moves 0,27 m in 2,6 s, so $v_2 = \frac{0.27}{2.6} \left[\frac{m}{s}\right] = 0{,}104\ m/s$

Determine the number of revolutions of each wheel. $n = revolutions / t$

Wheel 1

wheel 1 makes 3 revolutions in 2,6 s, so $n_1 = \frac{3}{2.6} \left[\frac{1}{s}\right] = 1{,}15\ s^{-1}$

Wheel 2

wheel 2 makes 3 revolutions in 2,6 s, so $n_2 = \frac{3}{2.6} \left[\frac{1}{s}\right] = 1{,}15\ s^{-1}$

<u>Conclusion:</u> In each measurement the power of the engine is constant, so we can conclude:

- when we change the diameter of the wheels, the number of revolutions *remains constant* .

- the larger the diameter of the wheels, the *higher* the speed at which the weights are displaced.

The displacement speed of the weight is the same as the peripheral speed of any point on the wheel.
The peripheral speed of a point in a UCM is the displacement of that point made per time interval, in other words: the circumference of the circle (πD) multiplied by the number of revolutions (n).

$$v = \pi . D . n$$

with v peripheral speed in $[m/s]$
D diameter in $[m]$
n number of revolutions in $[s^{-1}]$

The speed at which the weights are displaced can also be expressed by the angle that is gone through. This speed is called the angular speed ω and is expressed in the number of radians per time interval or rad/s.

We know that a complete circle has an angle of 360 ° or 2π rad. If a point on a circle has a number of revolutions of 1 revolution per second $(n = 1 s^{-1})$, we have $\omega = 2\pi\ rad/s$. If $n = 2 s^{-1}$, then $\omega = 4\pi\ rad/s$. This leads us to the formula:

$$\boxed{\omega = 2\pi . n}$$

with ω *angular speed in* $[rad/s]$
n *number of revolutions in* $[s^{-1}]$

Now calculate the angular speed of each wheel. $\boxed{\omega = 2\pi . n}$

Wheel 1
Wheel 1 has $n_1 = 1{,}15\ s^{-1}$, so $\omega_1 = 2\pi . 1{,}15\ [rad . s^{-1}] = 7{,}23\ rad/s$

Wheel 2
Wheel 2 has a $n_2 = 1{,}15\ s^{-1}$, so $\omega_2 = 2\pi . 1{,}15\ [rad . s^{-1}] = 7{,}23\ rad/s$

Link: Now we look for the relation between the peripheral speed and the angular speed of each weight. Complete the table underneath.

	radius	peripheral speed	angular speed	relation
Wheel 1	0,0318	0,223	7,23	0,0308
Wheel 2	0,0148	0,104	7,23	0,0144

Relation v/ω equals the *radius* of the wheel.
This results in a new formula $\frac{v}{\omega} = R$. So the relation between the peripheral speed and the angular speed can be expressed through the formula underneath:

$$\boxed{v = \omega . R}$$

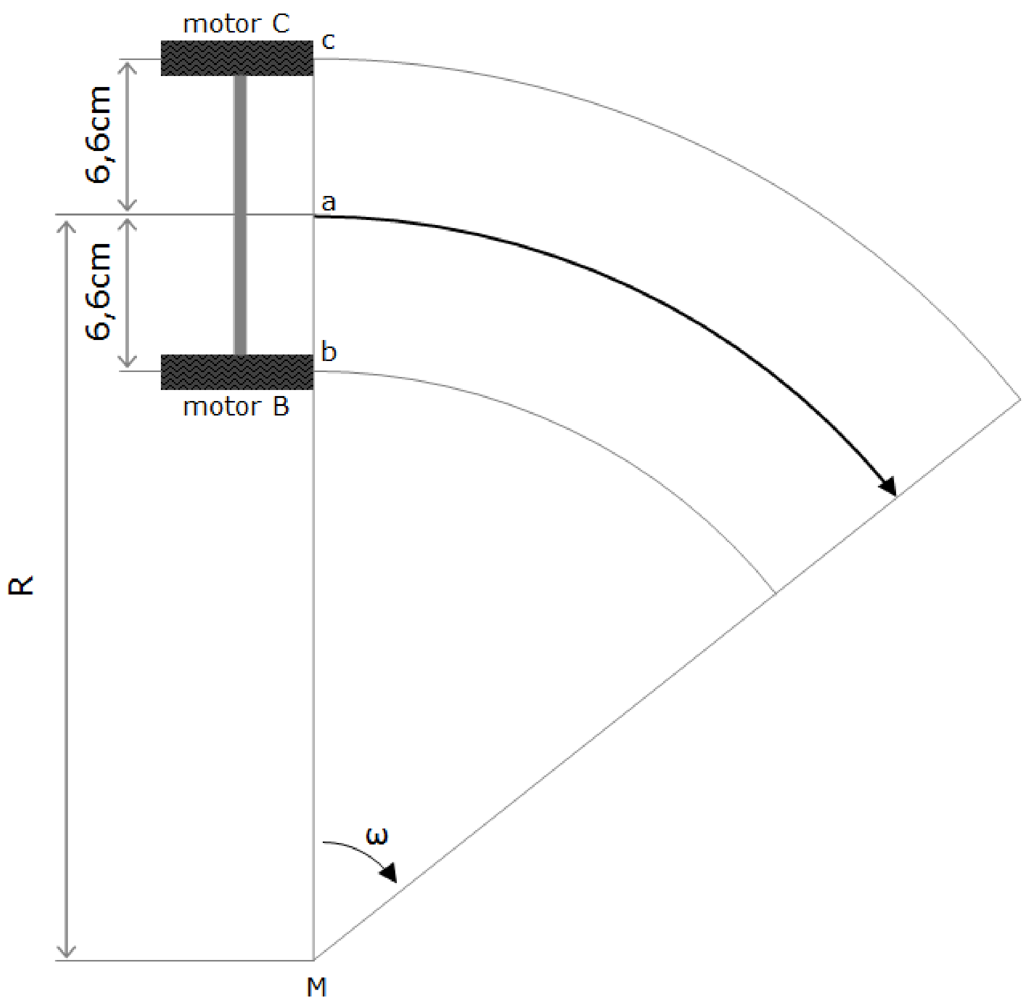
with v *peripheral speed in* $[m/s]$
ω *angular speed in* $[rad/s]$
R *radius in* $[m]$

Experiment for the pupils

Goal: In this task we have the robot drive in a circular motion. We want to be able to determine the size of the circle and the time the robot will need to cover this distance. So we will need to determine the speed of each wheel. To be able to test our solution we will need to convert the speed of the wheels into a power level we can program into the LEGO NXT. Calculations are done in cm.

Data: t = 15 s
R = 50 cm

Representation:



Solution:

a) Calculate the displacement of point a when it goes through a complete circle.

$$s_a = 2\pi R = 50cm \,.\, 2 \,.\, \pi = 314{,}16cm$$

Now calculate the peripheral speed of point a.

$$n = \frac{rotation}{t} = \frac{1}{15}[s^{-1}]$$
$$d = 2 \,.\, R = 2 \,.\, 50\ [cm] = 100\ [cm]$$
$$v = \pi . d . n = \pi \,.\, 100 \,.\, \frac{1}{15}\ [cm \,.\, s^{-1}] = 20{,}94\ cm/s$$

Calculate the angular speed of the NXT robot.

$$\omega = \frac{v_a}{R_a} = \frac{20{,}94\frac{cm}{s}}{50{,}00cm} = 0{,}419\ \frac{rad}{s}$$

Now you can easily calculate the speed of the wheels in points b and c.

$$v_b = \omega . R_b = 0{,}419\ .\ (50\ -\ 6{,}6)\ [rad/s\ .\ cm] = 18{,}2\ cm/s$$

$$v_c = \omega . R_c = 0{,}419\ .\ (50\ +\ 6{,}6)\ [rad/s\ .\ cm] = 23{,}7\ cm/s$$

b) From measurements we know that the relation between the speed and the power of an NXT can be expressed with the formula $v_{NXT} = 0{,}39 \,.\, P_{NXT} - 1{,}27$. Here v_{NXT} is the speed expressed in cm/s and P_{NXT} expresses the percentage of power of the NXT.

$$P_{NXT} = \frac{v_{NXT} + 1{,}27}{0{,}39}$$

$$P_{NXT_b} = \frac{v_b + 1{,}27}{0{,}39} = \frac{18{,}2 + 1{,}27}{0{,}39} = 49{,}9\ \cong\ 50\%$$

$$P_{NXT_c} = \frac{v_c + 1{,}27}{0{,}39} = \frac{23{,}7 + 1{,}27}{0{,}39} = 64{,}0\ \cong\ 64\%$$

c) Now try this out. Take a piece of chalk and draw a circle on the floor. Mark the start position. This way you can check whether the NXT describes a circle and stops at the start mark.

It can happen that the robot does not end at exactly the same spot it started at. This is influenced by the friction the NXT undergoes while in motion.